MAT262 - Image Processing - Cheat Sheet

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Filtering in the Frequency Domain

Series Fourier series represents a periodic function as a weighted sum of sines and cosines:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{j \frac{2\pi n}{T} t}$$

where the constants c_n (weights) are determined by orthogonality to be

$$c_n = \frac{1}{T} \int_{-T/2}^{T/2} f(t) e^{-j \frac{2\pi n}{T} t} dt$$

Transform The Fourier transform represents a function using all possible frequencies μ:

$$f(t) = \int_{-\infty}^{\infty} F(\mu) e^{j 2\pi \mu t} d\mu$$

Where $F(\mu)$ measures how much of μ the function $f(t)$ contains. The Fourier Transform is:

$$F(\mu) = \int_{-\infty}^{\infty} f(t) e^{-j 2\pi \mu t} dt$$

The function $F(\mu)$ is analogous to c_n for non-periodic functions.

DFT The DFT is a discrete counterpart of the Transform, representing a discrete function f_n as:

$$f_n = \frac{1}{M} \sum_{m=0}^{M-1} F_m e^{j 2\pi \frac{m}{M} n}$$

where F_m is the discrete Fourier transform:

$$F_m = \sum_{n=0}^{M-1} f_n e^{-j 2\pi \frac{n}{M} m}$$

Filter functions

Ideal A function with a jump. Fourier transform is the *sinc* function. Introduces ringing.

Butterworth The Butterworth filter is given by $H = \left[1 + (D/D_0)^{2n}\right]^{-1}$

Gaussian The Fourier transform of a Gaussian is a Gaussian. It is given by $H = \exp(-D^2/2D_0^2)$

Image restoration - Mean filters

All mean filters work in a neighborhood, with pixel values $x_1, x_2, ..., x_n$.

Arithmetic mean	Geometric mean	Harmonic mean
$a(x) = \frac{1}{n} \sum_i^n x_i$	$g(x) = \left[\prod_i^n x_i\right]^{\frac{1}{n}}$	$h(x) = \frac{n}{\sum_i^n \frac{1}{x_i}}$

Order statistics filters

In all filters: gather pixels, then sort, then...

Median, min, max Choose median, min or max value.

Alpha-trimmed mean Remove d extreme values, then compute the mean. If $d = 0$, it reduces to the arithmetic filter. If $d = n - 1$, we have the mean filter.

Adaptive filters

Noise Local noise reduction

$$\hat{f}(x, y) = g(x, y) - \frac{\sigma_\eta^2}{\sigma_\eta^2 + \sigma_L^2} \left[g(x, y) - m_L\right]$$

Median Adaptive median filter: The purpose is (1) remove impulse noise, (2) provide smoothing of other types of noise and (3) reduce distortion. The basic functionality is that if the pixel z_{xy} is impulse noise, replace it by median.

If it is not, leave it alone. Dynamic window size is employed if the value z_{xy} is either z_{min} or z_{max} (that is, it is itself an impulse).

Linear, position invariant degradation

Linear implies *additivity* and *homogeneity*. If the response of $H(u,v)$ to an impulse $\delta(t)$ is known, the response of any input is known.

Degradation model $g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$
$G(u,v) = H(u,v) \star F(u,v) + N(u,v)$
Where $h(x,y)$ is the degradation function, and $\eta(x,y)$ is the noise term.

Morphological image processing

In the following, A is a binary image and B is a structuring element.

Erosion $A \ominus B = \{z|(B)_z \subseteq A\}$
If entire structuring element B is contained inside A, use center of B.

Dilation $A \oplus B = \left\{z|(\hat{B})_z \cap A \neq \emptyset\right\}$
If the structuring element B and A has any overlap, use center of B.

Opening $A \circ B = (A \ominus B) \oplus B$
Rolling the structuring element on the inside boundary of A.

Closing $A \bullet B = (A \oplus B) \ominus B$
Rolling the structuring element on the outside boundary of A.

Hit-or-miss $A \circledast D = (A \ominus D) \cap \left[A^C \ominus (W - D)\right]$
Detects a pattern D in a binary image A.

Boundary Boundary extraction $\beta(A) = A - (A \ominus B)$

Hole filling $X_k = (X_{k-1} \oplus B) \cap A^C$

Connected Connected components: $X_k = (X_{k-1} \oplus B) \cap A$

Convex hull (smallest convex set), thinning and skeletons. They all use $\{B\} = \{B_1, B_2, ..., B_n\}$, a set of rotated structuring elements.

Edge detection

Marr-Hildreth (1) Filter with Gaussian lowpass, (2) detect edges using Laplacian convolution mask, (3) identify zero crossings(alternating signs).

Canny (1) Smooth with Guassian lowpass, (2) compute gradient magnitude and angle images (E.g. Sobel mask), (3) apply nonmaxima suppression (to thin edges), (4) double thresholding and connectivity analysis to detect and link edges(strong and weak edge pixels using 2 thresholds).

Image segmentation

Polygonal fit The iterative polygonal fit algorithm for regional segmentation uses divide and conquer to create a polygon corresponding to a sorted list of points in space.

Hough transform Parameter space, normal representation $x\cos\theta + y\sin\theta = \rho$, subdividing $\rho\theta$-space into accumulator cells and measure density.

Global thresh. A simple iterative algorithm is to select a threshold T, calculate averages m of the two classes defined by T, and set $T_k = \frac{1}{2}(m_1 + m_2)$ until convergence.

Otsu's method Minimize intra-class variance. Uses only histogram. Optimization problem. Works for arbitrary number of classes(groups), but typically employed for 2 or 3 classes.

Variable thresh. Three methods are: image partitioning, local image properties (mean and std.dev), moving averages.

Multiresolution processing

Image Pyramid Blur and resample at each level. Compare to get prediction residual pyramid.

Subband coding Filter bank. Spectrum splitting properties. Perfect reconstruction filter if input $f(n)$ equals output $f(\hat{n})$ of filter bank. Orthogonality of filters. Split into approximation and detail.

Expansions Expand $f(x) = \sum_k \alpha_k \phi_k(x)$, where ϕ_k are scaling functions.

Wavelets Simplest is Haar. Scaling function ϕ characterizes *scale* of wavelet, wavelet function ψ characterizes *shape* of wavelet. $f(x) = \sum_k c(k)\phi_k(x) + \sum_j \sum_k d_j(k)\psi_{j,k}(x)$

For more information, please see:
Rafael C. Gonzalez. *Digital Image Processing*. 3rd ed. Pearson/Prentice Hall, 2008.